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ON EQUATIONS OF THE STATE OF STRESS IN A PLATE OF VARIABLE THICKNESS*

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Method of asymptotic integration of three-dimensional equations of the theory of elasticity is used to construct the internal state of stress in a plate of variable thickness /1/. It is shown that it can generally be described by a system of differential equations of eighth order in the components of the displacement vector of the points of a plane projected inside the plate, with the equations of flexure and of plane state of stress not separated. In a particular case when the face surfaces are symmetrical with respect to this plane, the flexure and the plane state of stress are described by separate equations. The accuracy of the equations obtained is of the order of square of relative thickness of the plate away from the edge and other distortion lines of the stress state.

The boundary layer is not considered and conditions at the plate edge are not formulated.

1. We regard as a plate of variable thickness, a prismatic body (referred to the Cartesian xyz coordinate system) bounded by a cylindrical surface with the generatrix parallel to the z-axis (the edge), and two face surfaces $z = f_i(x, y)$ $(i = 1, 2)$.

We introduce the following assumptions.

1) $f_1(x, y) \geqslant 0$, $f_2(x, y) \leqslant 0$ everywhere in the region occupied by the body;

2) the thickness $2h(x, y) = f_1(x, y) - f_2(x, y)$ of the plate is small compared with any characteristic plane dimension L of the plate;

3) the face surfaces of the plate are sufficiently sloping so that $\partial f_i/\partial x$, $\partial f_i/\partial y \sim \varepsilon$ and $\varepsilon = (h_1 - h_2)/(2L)$ is a small parameter (h_1 and h_2 denote the smallest and largest half-thickness of the plate).

The conditions at the face surfaces can be written for an arbitrary surface load, in the form

$$z = f_i(x, y) \quad (i = 1, 2) \tag{1.1}$$

$$\sigma_{xx}\cos(n_i, x) + \sigma_{xy}\cos(n_i, y) + \sigma_{xz}\cos(n_i, z) = p_{ix} \quad (xy), \quad \sigma_{xz}\cos(n_i, x) + \sigma_{yz}\cos(n_i, y) + \sigma_{zz}\cos(n_i, z) = p_{iz}$$

Just as in the case of a plate of constant thickness /2/, we assume that

$$p_{ix} = \varepsilon^{-1} q_{ix} \quad (xy), \; p_{iz} = q_{iz} \quad (i = 1, 2) \tag{1.2}$$

where q_{ix}, q_{iy}, q_{iz} are independent of ε.

Using the above assumptions we write the equations of the face surfaces of the plate in the form

$$z = \varepsilon\lambda_i(x, y) \quad (i = 1, 2), \; \lambda_i(x, y) = \varepsilon^{-1} f_i(x, y), \quad \partial\lambda_i/\partial x, \partial\lambda_i/\partial y \sim \varepsilon^0 \tag{1.3}$$

In this case we can write

$$\cos(n_i, x) = -\varepsilon\, \partial\lambda_i/\partial x + O(\varepsilon^2) \quad (xy), \quad \cos(n_i, z) = 1 + O(\varepsilon^2) \tag{1.4}$$

Assuming that the stresses and displacements away from the edge do not vary rapidly with respect to the variables x and y and rapidly with respect to the variable z, we make the substitution $z = \varepsilon\zeta$ in the equations of equilibrium and the elasticity relations of the three-dimensional theory. We seek the solution of the resulting equations in the form

$$Q = \varepsilon^{-q} \sum_{s=0}^{S} \varepsilon^s Q^{(s)} \tag{1.5}$$

($q = 2$ for $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}, u, v$; $q = 1$ for σ_{xz}, σ_{yz}; $q = 0$ for σ_{zz}; $q = 3$ for W).

We obtain the following system of recurrence equations for the functions $Q^{(s)}$:

$$\frac{\partial\sigma_{xx}^{(s)}}{\partial x} + \frac{\partial\sigma_{xy}^{(s)}}{\partial y} + \frac{\partial\sigma_{xz}^{(s)}}{\partial\zeta} = 0 \quad (xy), \quad \frac{\partial\sigma_{xz}^{(s)}}{\partial x} + \frac{\partial\sigma_{yz}^{(s)}}{\partial y} + \frac{\partial\sigma_{zz}^{(s)}}{\partial\zeta} = 0$$

$$E\frac{\partial u^{(s)}}{\partial x} = \sigma_{xx}^{(s)} - \nu(\sigma_{yy}^{(s)} + \sigma_{zz}^{(s-2)}) \quad (xy), \quad E\frac{\partial W^{(s)}}{\partial\zeta} = \sigma_{zz}^{(s-4)} - \nu(\sigma_{xx}^{(s-2)} + \sigma_{yy}^{(s-2)})$$

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$$E\left(\frac{\partial u^{(s)}}{\partial \zeta}+\frac{\partial W^{(s)}}{\partial x}\right)=2(1+\nu)\sigma_{xz}^{(s-2)}\quad (xy),\qquad E\left(\frac{\partial u^{(s)}}{\partial y}+\frac{\partial v^{(s)}}{\partial x}\right)=2(1+\nu)\sigma_{xy}^{(s)}$$

Here and henceforth (xy) indicates that another equation exists obtained from the parent equation by replacing x by y and u by v.

Integrating the above equations with respect to ζ we obtain (for $s=0,1$)

$$W^{(s)}=w^{(s)}(x,y),\quad u^{(s)}=-\zeta\frac{\partial w^{(s)}}{\partial x}+\bar{u}^{(s)}(x,y)\quad (xy) \tag{1.6}$$

$$\sigma_{xx}^{(s)}=-\zeta\tau_{xx}^{(s)}+\bar{\sigma}_{xx}^{(s)}\quad (xy),\quad \sigma_{xy}^{(s)}=-\zeta\tau_{xy}^{(s)}+\bar{\sigma}_{xy}^{(s)},\quad \sigma_{xz}^{(s)}=\frac{\zeta^2}{2}\tau_{xz}^{(s)}-\zeta\bar{\sigma}_{xz}^{(s)}+\sigma_{xz}^{\circ(s)}\quad (xy)$$

$$\sigma_{zz}^{(s)}=-\frac{\zeta^3}{6}\tau_{zz}^{(s)}+\frac{\zeta^2}{2}\bar{\sigma}_{zz}^{(s)}-\zeta\sigma_{zz}^{\circ(s)}+\sigma_{zz}^{*(s)},\quad \tau_{xx}^{(s)}=\frac{E}{1-\nu^2}\left(\frac{\partial^2 w^{(s)}}{\partial x^2}+\nu\frac{\partial^2 w^{(s)}}{\partial y^2}\right)\quad (xy),\quad \tau_{xy}^{(s)}=\frac{E}{1+\nu}\frac{\partial^2 w^{(s)}}{\partial x\,\partial y}$$

$$\tau_{xz}^{(s)}=\frac{\partial\tau_{xx}^{(s)}}{\partial x}+\frac{\partial\tau_{xy}^{(s)}}{\partial y}=\frac{E}{1-\nu^2}\frac{\partial\Delta w^{(s)}}{\partial x}\quad (xy),\quad \tau_{zz}^{(s)}=\frac{\partial\tau_{xz}^{(s)}}{\partial x}+\frac{\partial\tau_{yz}^{(s)}}{\partial y}=\frac{E}{1-\nu^2}\Delta\Delta w^{(s)}$$

$$\bar{\sigma}_{xx}^{(s)}=\frac{E}{1-\nu^2}\left(\frac{\partial\bar{u}^{(s)}}{\partial x}+\nu\frac{\partial\bar{v}^{(s)}}{\partial y}\right)\quad (xy),\quad \bar{\sigma}_{xy}^{(s)}=\frac{E}{2(1+\nu)}\left(\frac{\partial\bar{u}^{(s)}}{\partial y}+\frac{\partial\bar{v}^{(s)}}{\partial x}\right)$$

$$\bar{\sigma}_{xz}^{(s)}=\frac{\partial\bar{\sigma}_{xx}^{(s)}}{\partial x}+\frac{\partial\bar{\sigma}_{xy}^{(s)}}{\partial y}=\frac{E}{2(1-\nu^2)}\left[(1-\nu)\Delta\bar{u}^{(s)}+(1+\nu)\frac{\partial\bar{\theta}^{(s)}}{\partial x}\right]\quad (xy)$$

$$\bar{\sigma}_{zz}^{(s)}=\frac{\partial\bar{\sigma}_{xz}^{(s)}}{\partial x}+\frac{\partial\bar{\sigma}_{yz}^{(s)}}{\partial y}=\frac{E}{1-\nu^2}\Delta\bar{\theta}^{(s)},\quad \sigma_{zz}^{\circ(s)}=\frac{\partial\sigma_{xz}^{\circ(s)}}{\partial x}+\frac{\partial\sigma_{yz}^{\circ(s)}}{\partial y},\quad \bar{\theta}^{(s)}=\frac{\partial\bar{u}^{(s)}}{\partial x}+\frac{\partial\bar{v}^{(s)}}{\partial y},\quad \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

From (1.6) we see that nine functions $Q^{(s)}$ sought are expressed by six unknown functions $\bar{u}^{(s)}$, $\bar{v}^{(s)}$, $w^{(s)}$, $\sigma_{xz}^{\circ(s)}$, $\sigma_{yz}^{\circ(s)}$, $\sigma_{zz}^{*(s)}$ of two variables x and y. To find these functions we turn to the conitions at the face surfaces of the plate. Substituting (1.2), (1.4) and (1.5) into the conditions (1.1) and equating the coefficients of like powers of ε in the left- and right-hand sides, we obtain the following expression for $s=0,1$ for $\zeta=\lambda_i(x,y)$ $(i=1,2)$

$$-\frac{\partial\lambda_i}{\partial x}\sigma_{xx}^{(s)}-\frac{\partial\lambda_i}{\partial y}\sigma_{xy}^{(s)}+\sigma_{xz}^{(s)}=p_{ix}^{(s)}\quad (xy),\quad -\frac{\partial\lambda_i}{\partial x}\sigma_{xz}^{(s)}-\frac{\partial\lambda_i}{\partial y}\sigma_{yz}^{(s)}+\sigma_{zz}^{(s)}=p_{iz}^{(s)} \tag{1.7}$$

$$p_{ix}^{(0)}=q_{ix},\quad P_{iy}^{(0)}=q_{iy},\quad p_{iz}^{(0)}=q_{iz},\quad p_{ix}^{(1)}=p_{iy}^{(1)}=p_{iz}^{(1)}=0\quad (i=1,2)$$

Let us now substitute into (1.7) the corresponding values from (1.6). Performing the necessary manipulations we obtain

$$\sigma_{xz}^{\circ(s)}=\frac{1}{2}\left\{(p_{1x}^{(s)}+p_{2x}^{(s)})-\frac{\partial}{\partial x}[(\delta^2+H^2)\tau_{xx}^{(s)}-2\delta\bar{\sigma}_{xx}^{(s)}]-\frac{\partial}{\partial y}[(\delta^2+H^2)\tau_{xy}^{(s)}-2\delta\bar{\sigma}_{xy}^{(s)}]\right\}\quad (xy) \tag{1.8}$$

$$\sigma_{zz}^{*(s)}=\frac{1}{2}(p_{1z}^{(s)}+p_{2z}^{(s)})+\frac{1}{6}\frac{\partial}{\partial x}[\delta(\delta^2+3H^2)\tau_{xz}^{(s)}-3(\delta^2+H^2)\bar{\sigma}_{xz}^{(s)}+6\delta\sigma_{xz}^{\circ(s)}]+\frac{1}{6}\frac{\partial}{\partial y}[\delta(\delta^2+3H^2)\tau_{yz}^{(s)}-3(\delta^2+H^2)\bar{\sigma}_{yz}^{(s)}+6\delta\sigma_{yz}^{\circ(s)}]$$

$$\frac{\partial}{\partial x}[H(\delta\tau_{xx}^{(s)}-\bar{\sigma}_{xx}^{(s)})]+\frac{\partial}{\partial y}[H(\delta\tau_{xy}^{(s)}-\bar{\sigma}_{xy}^{(s)})]=\frac{1}{2}(p_{1x}^{(s)}-p_{2x}^{(s)}) \tag{1.9}$$

$$\frac{\partial}{\partial x}[H(\delta\tau_{xy}^{(s)}-\bar{\sigma}_{xy}^{(s)})]+\frac{\partial}{\partial y}[H(\delta\tau_{yy}^{(s)}-\bar{\sigma}_{yy}^{(s)})]=\frac{1}{2}(p_{1y}^{(s)}-p_{2y}^{(s)}),\quad \frac{\partial^2}{\partial x^2}(H^3\tau_{xx}^{(s)})+2\frac{\partial^2}{\partial x\,\partial y}(H^3\tau_{xy}^{(s)})+\frac{\partial^2}{\partial y^2}(H^3\tau_{yy}^{(s)})+$$

$$3H\frac{\partial^2\delta}{\partial x^2}(\delta\tau_{xx}^{(s)}-\bar{\sigma}_{xx}^{(s)})+6H\frac{\partial^2\delta}{\partial x\,\partial y}(\delta\tau_{xy}^{(s)}-\bar{\sigma}_{xy}^{(s)})+3H\frac{\partial^2\delta}{\partial y^2}(\delta\tau_{yy}^{(s)}-\bar{\sigma}_{yy}^{(s)})=\frac{3}{2}\Big\{(p_{1z}^{(s)}-p_{2z}^{(s)})-$$

$$\frac{\partial}{\partial x}[H(p_{1x}^{(s)}+p_{2x}^{(s)})]-\frac{\partial}{\partial y}[H(p_{1y}^{(s)}+p_{2y}^{(s)})]-\frac{\partial\delta}{\partial x}(p_{1x}^{(s)}-p_{2x}^{(s)})-\frac{\partial\delta}{\partial y}(p_{1y}^{(s)}-p_{2y}^{(s)})$$

$$(2H=\lambda_1-\lambda_2,\quad 2\delta=\lambda_1+\lambda_2)$$

We see from (1.8) and (1.6) that the functions $\sigma_{xz}^{\circ(s)}$, $\sigma_{yz}^{\circ(s)}$ and $\sigma_{zz}^{*(s)}$ are directly expressed in terms of the functions $\bar{u}^{(s)}$, $\bar{v}^{(s)}$, $w^{(s)}$, and the solution of the problem of the state of stress in a plate of variable thickness is reduced to solving three equations (1.9).

Let us now limit ourselves to the first two terms in (1.5), combine the equations (1.9) written for $s=0,1$ by expressing the stresses in terms of the displacements in accordance with (1.6), and return to the initial notation of the problem. This yields the following system of equations for solving the state of stress in a plate of variable thickness:

$$2\frac{\partial}{\partial x}\left\{h\left[\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}-\varphi\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]\right\}+(1-\nu)\frac{\partial}{\partial y}\left\{h\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2\varphi\frac{\partial^2 w}{\partial x\,\partial y}\right]\right\}=-\frac{1-\nu^2}{E}(q_{1x}-q_{2x}) \quad (1.10)$$

$$(1-\nu)\frac{\partial}{\partial x}\left(h\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2\varphi\frac{\partial^2 w}{\partial x\,\partial y}\right]\right\}+2\frac{\partial}{\partial y}\left\{h\left[\frac{\partial v}{\partial y}+\nu\frac{\partial u}{\partial x}-\varphi\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)\right]\right\}=-\frac{1-\nu^2}{E}(q_{1y}-q_{2y})$$

$$D\Delta\Delta w+2\frac{\partial D}{\partial x}\frac{\partial\Delta w}{\partial x}+2\frac{\partial D}{\partial y}\frac{\partial\Delta w}{\partial y}+\frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+$$

$$\frac{2Eh}{1-\nu^2}\left\{\frac{\partial^2\varphi}{\partial x^2}\left[\varphi\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)-\left(\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right)\right]+\frac{\partial^2\varphi}{\partial y^2}\left[\varphi\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)-\left(\frac{\partial v}{\partial y}+\nu\frac{\partial u}{\partial x}\right)\right]+\right.$$

$$\left.(1-\nu)\frac{\partial^2\varphi}{\partial x\,\partial y}\left[2\varphi\frac{\partial^2 w}{\partial x\,\partial y}-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]\right\}=q_{1z}-q_{2z}-\frac{\partial}{\partial x}[h(q_{1x}+q_{2x})]-\frac{\partial}{\partial y}[h(q_{1y}+q_{2y})]-$$

$$\frac{\partial\varphi}{\partial x}(q_{1x}-q_{2x})-\frac{\partial\varphi}{\partial y}(q_{1y}-q_{2y}),\quad D=\frac{2Eh^3}{3(1-\nu^2)},\quad 2h=f_1-f_2=2H\varepsilon,\quad 2\varphi=f_1+f_2=2\delta\varepsilon$$

Here $z=\varphi(x,y)$ is the equation of the "middle" surface of the plate (the distance separating the points of this surface from the face surfaces of the plate are measured in the direction of the z-axis).

From (1.10) it follows that in the general case the state of stress in a plate of variable thickness is described by a system of three, eighth order differential equations in terms of the components of the displacement vector of the points of the xy-plane, with the equations of flexure and plane state of stress not separated from each other. The accuracy of the equations is of the order of ε^2 compared with unity.

2. We consider the tangential forces N_x, N_y, N_{xy}, intersecting the forces Q_x and Q_y and the moments M_x, M_y, M_{xy}

$$N_x=\int_{f_1}^{f_2}\sigma_{xx}\,dz=\frac{2Eh}{1-\nu^2}\left[\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}-\varphi\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)\right]\quad (xy)$$

$$N_{xy}=\int_{f_1}^{f_2}\sigma_{xy}\,dz=\frac{Eh}{1+\nu}\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}-2\varphi\frac{\partial^2 w}{\partial x\,\partial y}\right],\quad M_x=\int_{f_1}^{f_2}z\sigma_{xx}\,dz=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\varphi N_x\quad (xy)$$

$$M_{xy}=\int_{f_1}^{f_2}z\sigma_{xy}\,dz=-D(1-\nu)\frac{\partial^2 w}{\partial x\,\partial y}+\varphi N_{xy},\quad Q_x=\int_{f_1}^{f_2}\sigma_{xz}\,dz=\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}+f_1q_{1x}-f_2q_{2x}\quad (xy)$$

The solution equations (1.10) can not be written in the form

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=-(q_{1x}-q_{2x})\quad (xy),\quad \frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=q_{1z}-q_{2z} \quad (2.1)$$

When the face surfaces of the plate are distributed symmetrically about the xy-plane, we have $\varphi(x,y)=0$ and equations (1.10) separate into two independent systems

$$2\frac{\partial}{\partial x}\left[h\left(\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right)\right]+(1-\nu)\frac{\partial}{\partial y}\left[h\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]=-\frac{1-\nu^2}{E}(q_{1x}-q_{2x}) \quad (2.2)$$

$$(1-\nu)\frac{\partial}{\partial x}\left[h\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+2\frac{\partial}{\partial y}\left[h\left(\frac{\partial v}{\partial y}+\nu\frac{\partial u}{\partial x}\right)\right]=-\frac{1-\nu^2}{E}(q_{1y}-q_{2y})$$

$$D\Delta\Delta w+2\frac{\partial D}{\partial x}\frac{\partial\Delta w}{\partial x}+2\frac{\partial D}{\partial y}\frac{\partial\Delta w}{\partial y}+\frac{\partial^2 D}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+ \quad (2.3)$$

$$\frac{\partial^2 D}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y}=q_{1z}-q_{2z}-\frac{\partial}{\partial x}[h(q_{1x}+q_{2x})]-\frac{\partial}{\partial y}[h(q_{1y}+q_{2y})]$$

Equations (2.2) describe the plane state of stress in the plate, and (2.3) its flexure. Equations (2.3) is identical to the equation of the classical theory. Equations (1.10) can be used to obtain the equations of the state of stress in a plate of constant thickness $2h$ with small initial distortion of the middle surface. Let the initially distorted middle surface of the plate be described by $z=\varphi_0(x,y)$. Then the equations of the face surfaces of the plate will be

$$z=f_i(x,y)=\varphi_0(x,y)\pm h\quad (i=1,2)$$

and equations (1.10) now become

$$(1-\nu)\Delta u+(1+\nu)\frac{\partial\theta}{\partial x}-2\varphi_0\frac{\partial\Delta w}{\partial x}-2\frac{\partial\varphi_0}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)-2(1-\nu)\frac{\partial\varphi_0}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y}=-\frac{1-\nu^2}{E}(q_{1x}-q_{2x})$$

$$(1-\nu)\Delta v+(1+\nu)\frac{\partial\theta}{\partial y}-2\varphi_0\frac{\partial\Delta w}{\partial y}-2\frac{\partial\varphi_0}{\partial y}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)-2(1-\nu)\frac{\partial\varphi_0}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y}=-\frac{1-\nu^2}{E}(q_{1y}-q_{2y})$$

$$D\Delta\Delta w+\frac{2Eh}{1-\nu^2}\left\{\frac{\partial^2\varphi_0}{\partial x^2}\left[\varphi_0\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)-\right.\right.$$

$$\left.\left(\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right)\right]+\frac{\partial^2\varphi_0}{\partial y^2}\left[\varphi_0\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)-\left(\frac{\partial v}{\partial y}+\nu\frac{\partial u}{\partial x}\right)\right]+(1-\nu)\frac{\partial^2\varphi_0}{\partial x\,\partial y}\left.\left[2\varphi_0\frac{\partial^2 w}{\partial x\,\partial y}-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]\right\}=$$

$$q_{1z}-q_{2z}-h\left[\frac{\partial}{\partial x}(q_{1x}+q_{2x})+\frac{\partial}{\partial y}(q_{1y}+q_{2y})\right]-\frac{\partial\varphi_0}{\partial x}(q_{1x}-q_{2x})-\frac{\partial\varphi_0}{\partial y}(q_{1y}-q_{2y})$$

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